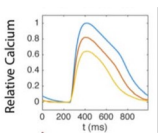
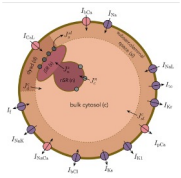
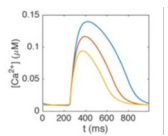
Computational inversion of optical data from cardiac MPS

SIMBER On-Site Kick-Off

Nick Forsch, Henrik Finsberg







Overview of the workshop

***Goal**: Understand the concepts involved with the inversion of MPS data for predicting drug targets and effects*

BACKGROUND	INVERSION IN MPS	DEMONSTRATION USING GRID SEARCH INVERSION
• What is computational modeling? • The forward problem in modeling • The inverse problem • Examples in cardiac modeling with clinical applications	• Steps of the inversion framework • Limitations and challenges	• Explanation of algorithm • Hands-on demo with synthetic data • Hands-on demo with real data

These are not my ideas

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What is computational modeling?

The process of simulating physical, real-world systems using mathematics, physics, and computer science.

The bulk of the work:

- Constructing representations of systems using mathematical expressions
- Developing a computational framework that can simulate the system in time and/or space
- Validating the model to improve its accuracy and applicability

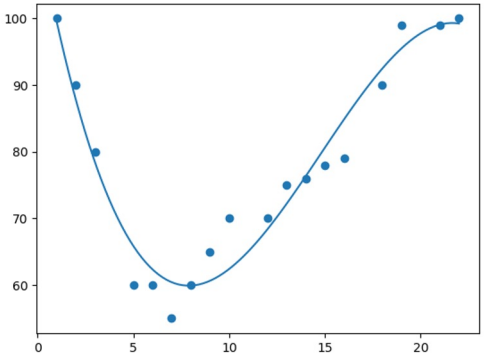
Important concepts in computational modeling

Model fidelity

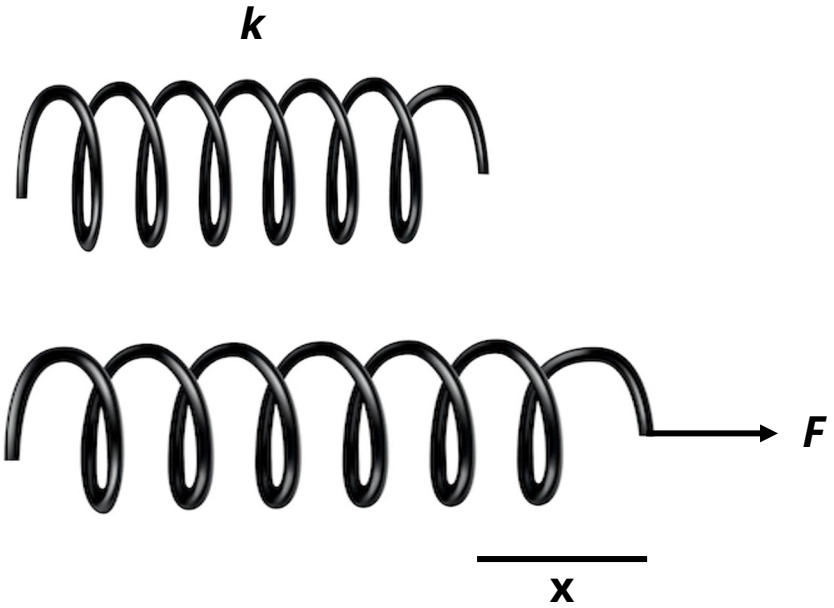
- How well does the model represent the state and behavior of the system?
- Model may be suitable for studying a disease state but not a healthy state (or vice versa)

Data assimilation

- Estimating parameters of a mathematical model by fitting to data
- Most use of models involve this step since the mathematical expressions (the theory) is already determined



The forward problem in modeling



Hooke's Law:

$$F = kx$$

Observables: F, x

Parameters: k

If the material constant k is known, then by knowing x we can solve for F, or vice versa.

The inverse problem in modeling

What happens when we can only observe the force and the displacement of the spring and we know the underlying theory relating the observables?

For Hooke's Law, this is simple: $F/x = k$

The inverse problem is the process of calculating the factors that produce an observable set of information using a given model

The inverse problem in modeling

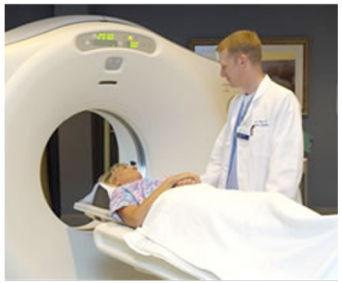
Cause → **Effect** is how the physical world typically operates

- We are given the effect (some observations) and we want to reveal the causes, both quantitatively and qualitatively
- Examples
  - Based on observable weather patterns (temperatures, wind speeds, precipitation, etc.) what are the causes (heat sources, sinks, atmospheric effects, etc.) so that we can make accurate forecasts?
  - If we want to launch a satellite into the atmosphere with a specific trajectory and orbit, what engine thrust magnitude and direction is needed?

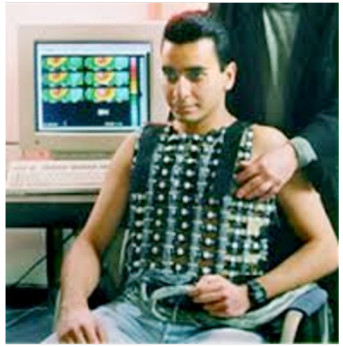
Examples of inverse problems with clinical applications

Measurements are clinically viable

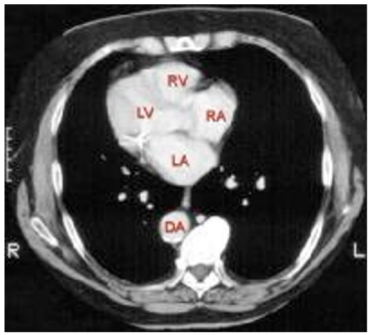
Imaging



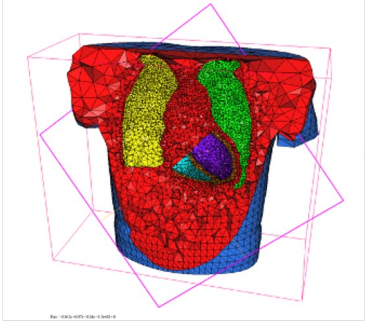
Body surface electrodes



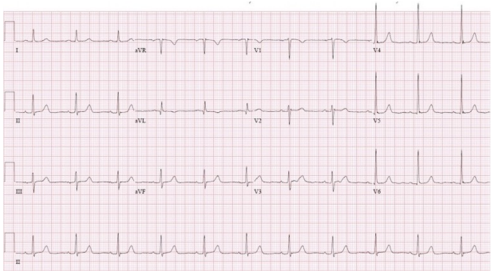
Segmentation



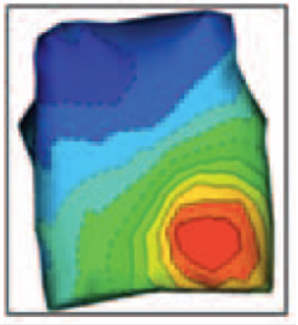
Meshing



Surface electrocardiogram



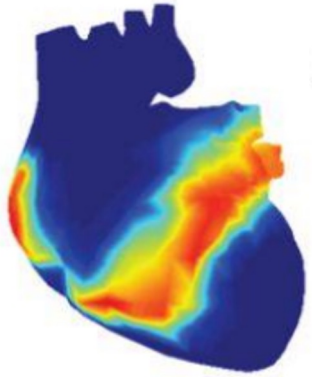
Body surface potentials

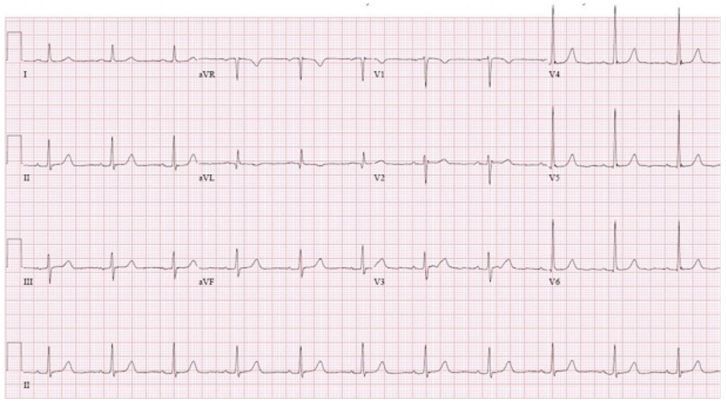


The inverse problem in cardiac electrophysiology

Cardiac electrophysiology: our model is 3D electrical propagation during an single beat in a 4-chamber geometry

- The forward problem: using our model, simulate an infarct in a specific region of the heart and compute what the resultant surface ECG would look like

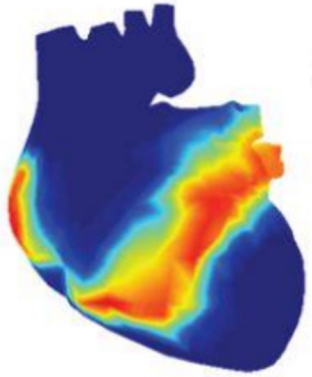


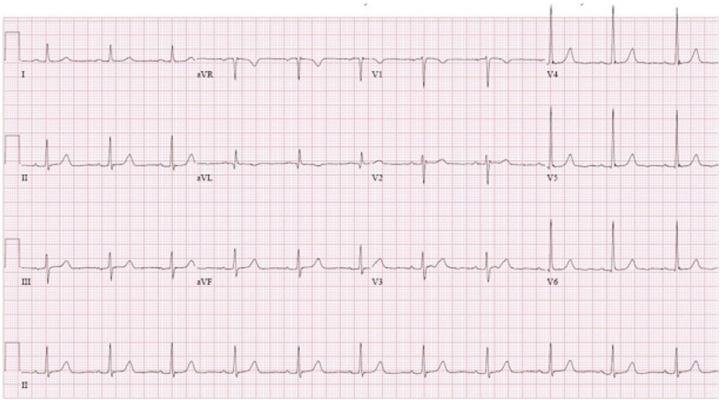


The inverse problem in cardiac electrophysiology

Cardiac electrophysiology: our model is 3D electrical wave propagation during an single beat in a 4-chamber geometry

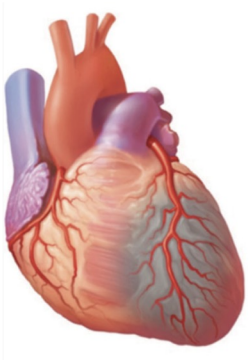
- The inverse problem: given a measured ECG, determine what infarct size and location would have produced the ECG (*more clinically realistic*)





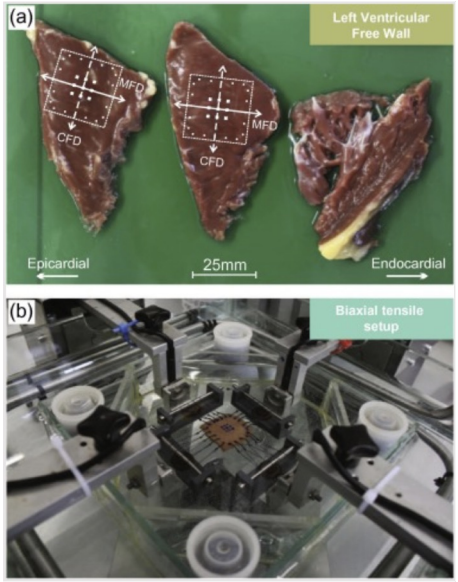
Assessing myocardial scarring in an infarcted heart

After myocardial infarction, tissue death leads to scarring and a change in material properties of the heart



A stiffer, scarred muscle does not function as well as the undamaged, healthy muscle

Material properties of the myocardium can be assessed through biaxial experiments



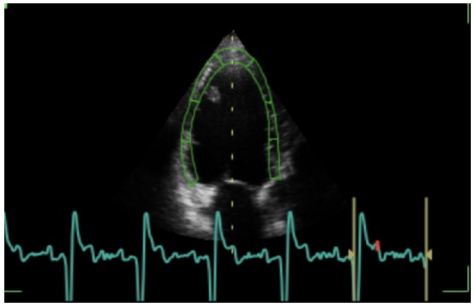
Sommer, G. et al, 2015. Biomechanical properties and microstructure of human ventricular myocardium. Acta biomaterialia, 24, pp.172-192.

How do we determine the properties of the scarred heart *in vivo*?

The inverse problem in cardiac biomechanics

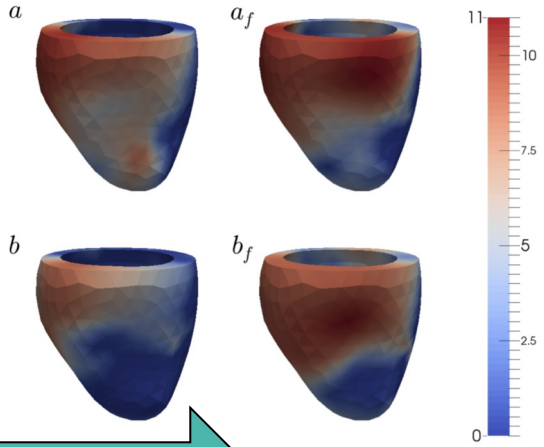
How do we determine the properties of the scarred heart *in vivo*?

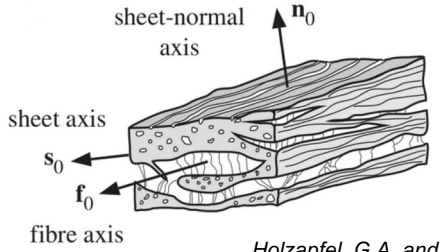
Measurements/observables



Material model

	Contribution from	Extracellular matrix
+	Contribution from	Fibers
+	Contribution from	Sheets
+	Contribution from	Angle between fibers and sheets





Relationship between material and observables

$$\sigma = J^{-1}\frac{\partial \Psi}{\partial \mathbf{F}}\mathbf{F}^T$$

Data assimilation

(a few other things...)

Holzapfel, G.A. and Ogden, R.W., 2009. Philosophical Transactions of the Royal Society of London A: Mathematical, Physical and Engineering Sciences

Common challenges with forward and inverse problems

1. Measurement error and uncertainty → inversion error and uncertainty
2. Model fidelity
3. Underdetermined systems (the solution is non-unique)
4. Validation and verification of the solution to inverse problems

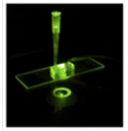
Inversion problems in cardiac microphysiological systems

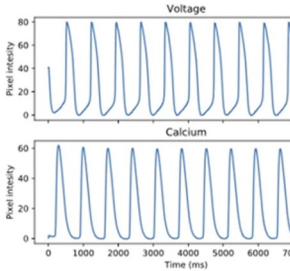
In vitro observation

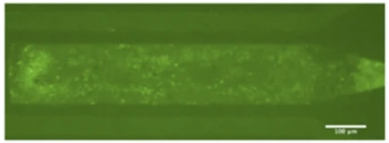
Computational inversion

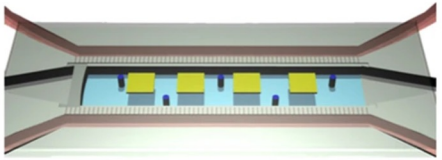
Predicting drug mechanism in mature cells

Optical Action Potential Measurements

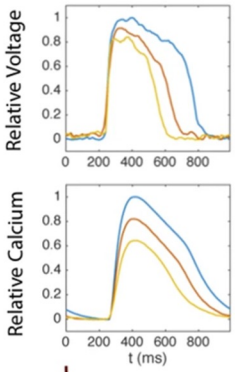




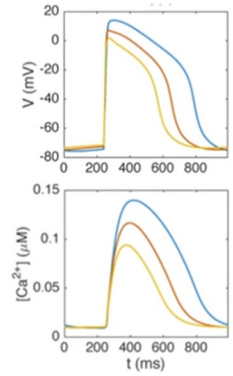


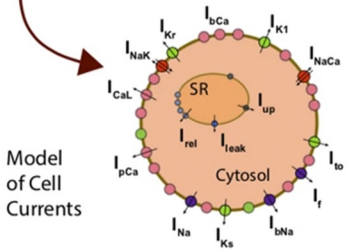


Measurements Under Drug



Optimized Mathematical Model



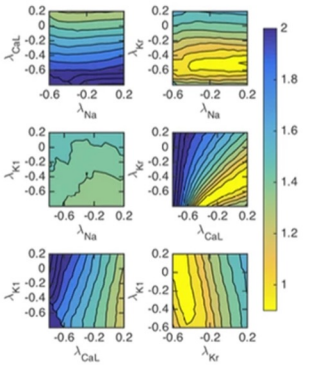


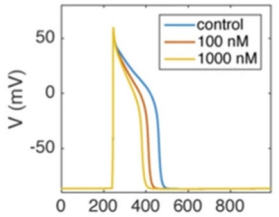
Base Model

$$v_t = -\sum_i q_i I_i(v,s)$$

Drug Model

$$v_t = -\sum (1+\lambda_i) q_i I_i(v,s)$$

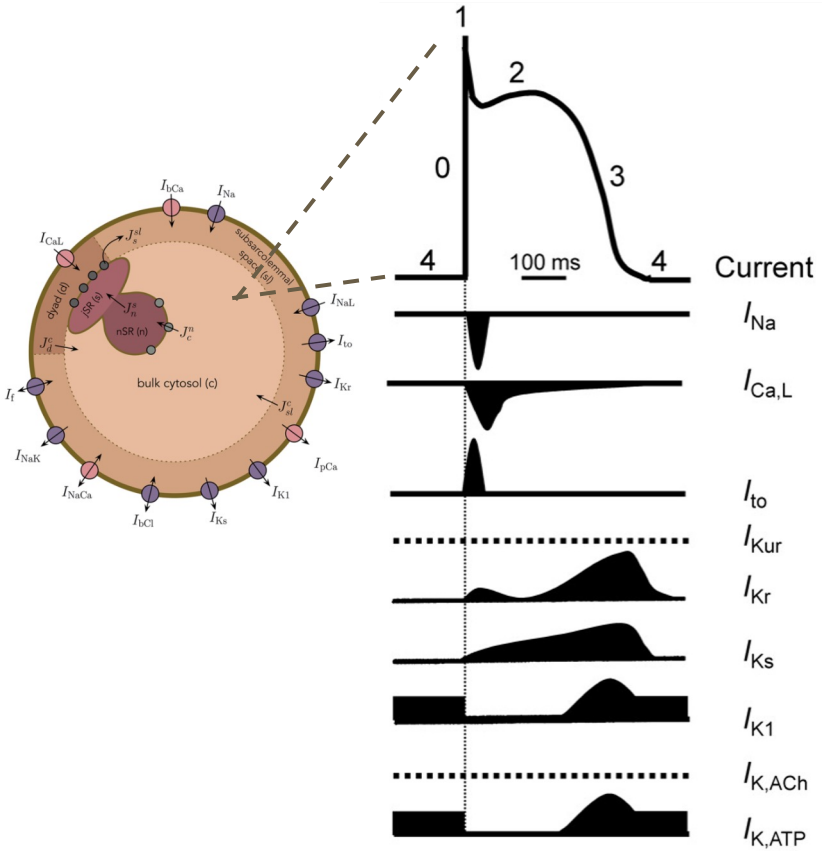


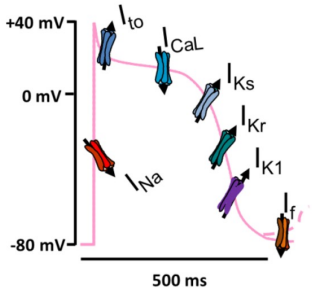


Mapping Effect in Mature Cells

Tveito *et al*, *Sci Rep*, 2018

MPS forward problem





The standard model of the membrane potential of an excitable cell:

$$\frac{dv}{dt} = -\frac{1}{C_m}\sum_x N_x i_x$$

For voltage-gated ion channels, the average single channel current:

$$i_x = g_x o_x (v - E_x)$$

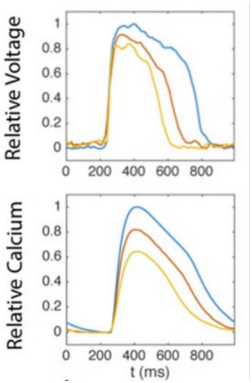
We parameterize the model by scaling the default conductance, g_x^0 :

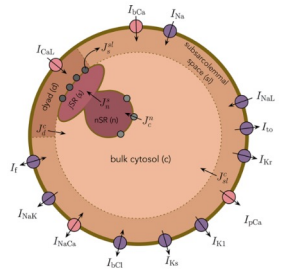
$$g_x^C = (1 + \lambda_x^C) g_x^0 \qquad g_x^D = (1 + \lambda_x^D) g_x^0$$

MPS inverse problem

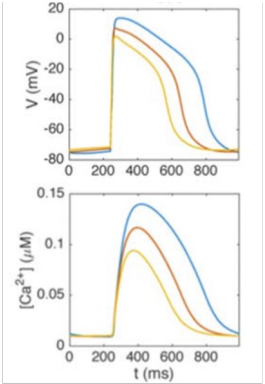
- The inverse problem (i.e. inversion) is the process of estimating parameters of our biophysical mathematical model such that the outputs match the measured inputs (relative voltage and calcium)
- Through inversion we can reveal the relative change in the parameters of a mathematical cell AP model due to the introduction of a drug
- Differences in cell model parameters (e.g. maximal ionic channel conductances) represent a block or promotion of a specific ionic current

Measured inputs





Model outputs



The first step: Determine if the problem is solvable

$$f = x_1 + x_2$$

The concept of “well-posed” problems:

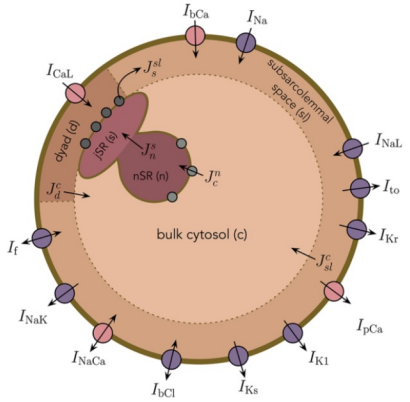
- The solution exists
- The solution is unique

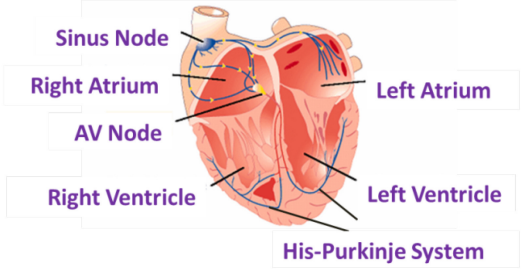
The concept of “well-conditioned” problems

- Small changes to the input lead to small changes to the output

>>> MPS inversion is <u>often</u> ill-posed and ill-conditioned

Second step: construct or adapt a parameterized model





http://www.scholarpedia.org/article/Models_of_cardiac_cell

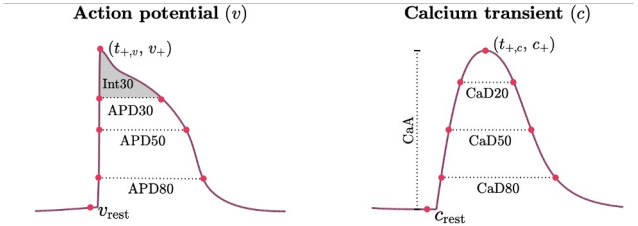
Third step: Select an optimization method for fitting the model to data

Iterative methods

- Newton's method
- Gradient descent
- Pattern search

Heuristic methods

- Differential evolution
- Genetic algorithm
- Particle swarm optimization



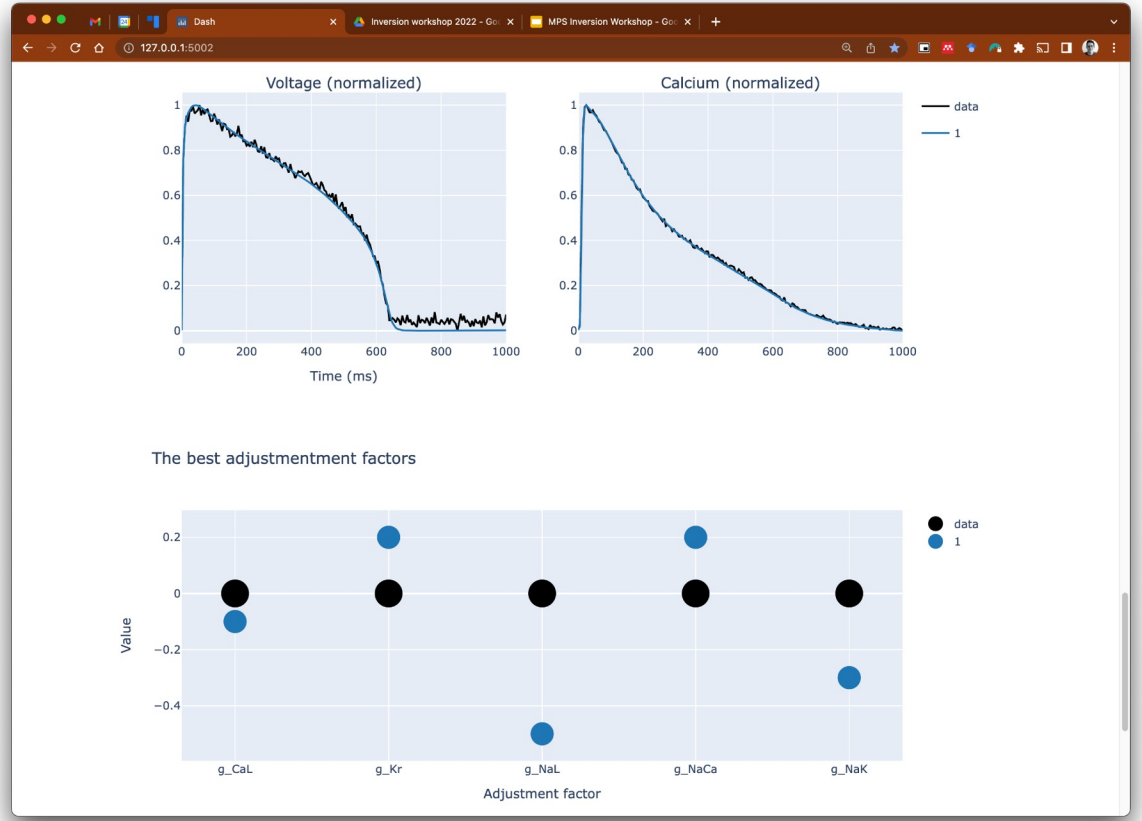
Grid search optimization

- Pre-compute many possible solutions by running the cell model with random or structured parameter sets
- Define features of the model output and measured input that can be used to quantify errors
- For data you want to invert, compute a cost function value for every point in your grid
- Sort all points in the grid by cost function and analyze the top *N* parameter sets

Cost function

$$J(\lambda) = \sum_i w_{i,v} \left(f_{i,v}^{\text{model}}(\lambda) - f_{i,v}^{\text{data}}(\lambda) \right)^2 + w_{i,c} \left(f_{i,c}^{\text{model}}(\lambda) - f_{i,c}^{\text{data}}(\lambda) \right)^2$$

Final step: solve the inverse problem



Demonstration of inversion using grid search optimization

- Demonstration of MPS inversion using synthetic data: `python app.py`
- Demonstration using real data: `python app_real_data.py`